UNIVERSITY OF MANITOBA

DATE: April 15, 2013 — FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Engineering Mathematical Analysis 2 — TIME: 3 hours

COURSE: MATH 2132 — EXAMINER: G.I. Moghaddam & M. Virgilio

NAME: (Print in ink) ______________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________

(I understand that cheating is a serious offense)

Place a check-mark (✓) against your instructor's name.

❑ A01 G.I. Moghaddam ❑ A02 M. Virgilio

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 2 blank pages for rough work together with a *formulas sheet*. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	6	
2	11	
3	8	
4	7	
5	10	
6	9	
7	9	
8	8	
9	8	
10	10	
11	14	
Total:	100	

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[6] 1. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n\,(2n)!\,[1 \cdot 7 \cdot 13 \cdot 19 \cdots (6n+1)]}{3^n\,(3n)!}\,(x-5)^{3n}.$$

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[11] 2. Find the Taylor series about 2 for the function

$$f(x) = \left(\frac{x-2}{x-1}\right)^2 .$$

Express your answer in sigma notation, simplify as much as possible, and find the open interval of convergence. Then use your answer to find the sum of $\displaystyle\sum_{n=2}^{\infty} \frac{n-1}{2^n}$.

[4] 3. (a) Evaluate the following integral using infinite series

$$\int_0^1 x^2 \cos(\sqrt{x})\, dx\,.$$

Express your answer in sigma notation.

[4] (b) If you truncate the series in part (a) after the **third** term, what is a maximum possible error? Explain why you can claim that your answer is a maximum error.

[7] 4. Find, in **implicit** form, a one parameter family of solutions for the differential equation

$$\frac{dy}{dx} = \frac{xy + 2y - x - 2}{xy - 3y + x - 3}.$$

[10] 5. Find the solution of the initial value problem

$$-3y'' = 2x\,(y')^4\,, \quad y'(1) = 1\,, \quad y(1) = 10.$$

[9] 6. Find a general solution for

$$y''' - 4y'' = 24\,x\,.$$

[9] 7. Consider the initial value problem

$$\frac{dA}{dt} = kA\,, \quad A(0) = A_0\,, \quad k < 0$$

as the model for the decay of a radioactive substance, where $A(t)$ is the amount of the radioactive substance present, A_0 is the initial amount of the radioactive substance and k is a constant.

(a) Solve the differential equation and show that, in general, the half-life T is $T = -\dfrac{\ln 2}{k}$ (i.e. the time it will take to get $A(t) = \dfrac{1}{2}A_0$).

(b) Show that the solution of the initial value problem in part (a) can be written as $A(t) = A_0\, 2^{-\frac{t}{T}}$.

(c) How long will it take for the radioactive substance to decay to $\dfrac{1}{8}$ of its initial amount?

[8] 8. Given that $m^2\,(m-2)^2 = 0$ is the auxiliary equation of the homogeneous differential equation associated with the linear differential equation

$$\phi(D)y = (x + x^3)\,e^{2x} + 1\,.$$

(a) Find the general solution of $\phi(D)y = 0$.

(b) What is the **form** of a particular solution $y_p(x)$ of the above nonhomogeneous differential equation?
DO NOT EVALUATE THE COEFFICIENTS IN $y_p(x)$.

[8] 9. Find the Laplace transform of the function

$$f(t) = \begin{cases} e^{-t} & \text{if} \quad 0 \le t < 2\,, \\ t^2 & \text{if} \quad t \ge 2\,. \end{cases}$$

[10] 10. Find $\mathscr{L}^{-1}\left\{\dfrac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}$.

[14] 11. Use Laplace transforms to solve the initial-value problem

$$y'' + 9\,y \;=\; 3\,\delta(t-\pi)\; + 18\,\mathscr{U}(t-2)\,, \qquad y(0) = 1, \qquad y'(0) = 10.$$

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ANSWERS

Q1 $R_x = \dfrac{3}{2}$ and $\dfrac{7}{2} < x < \dfrac{13}{2}$.

Q2 $f(x) = \displaystyle\sum_{n=1}^{\infty} n(-1)^{n+1}(x-2)^{n+1}$ if $1 < x < 3$.

Then put $x = \dfrac{3}{2}$ to get $\displaystyle\sum_{n=2}^{\infty} \frac{n-1}{2^n} = 1$.

Q3-a $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+3)(2n)!}$

Q3-b It is an alternating series with $b_n = \dfrac{1}{(n+3)(2n)!}$ and the maximum possible error is $b_3 = \dfrac{1}{6\,(6!)} = \dfrac{1}{4320}$.

Q4 $y + 2\ln|y-1| = x + 5\ln|x-3| + C$.

Q5 $y = 3\sqrt[3]{x} + 7$.

Q6 $y_h(x) = C_1 + C_2x + C_3e^{4x}$ and $y_p(x) = -x^3 - \dfrac{3}{4}x^2$ so then
$y = y_h + y_p = C_1 + C_2x + C_3e^{4x} - x^3 - \dfrac{3}{4}x^2$.

Q7-a Take integral and use $A(t) = \dfrac{1}{2}A_0$ to get $T = -\dfrac{\ln 2}{k}$.

Q7-b $A(t) = A_0\, e^{-\frac{t}{T}}$

Q7-c $t = \dfrac{-3\ln 2}{k}$.

Q8-a $y_h(x) = C_1 + C_2x + (C_3 + C_4x)e^{2x}$.

Q8-b $y_p(x) = B_1x^5e^{2x} + B_2x^4e^{2x} + A_1x^3e^{2x} + A_2x^2e^{2x} + Dx^2$.

Q9 $\dfrac{1}{s+1} - \dfrac{e^{-2(s+1)}}{s+1} + e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{4}{s}\right)$.

Q10 $-e^{-t} + 3e^t\cos 2t + 8e^t\sin 2t$.

Q11 $y(t) = \cos 3t + \dfrac{10}{3}\sin 3t + \sin 3(t-\pi)u(t-\pi) + 2u(t-2) - 2\cos 3(t-2)u(t-2)$.

Formulas Sheet	
Function: f(t)	**Laplace transform**: $\mathscr{L}\{\mathbf{f(t)}\} = \mathbf{F(s)}$
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$, (n is a positive integer)
$\sin kt$	$\dfrac{k}{s^2+k^2}$
$\cos kt$	$\dfrac{s}{s^2+k^2}$
$t\ \sin kt$	$\dfrac{2ks}{(s^2+k^2)^2}$
$t\ \cos kt$	$\dfrac{s^2-k^2}{(s^2+k^2)^2}$
e^{at}	$\dfrac{1}{s-a}$
$t^n\, e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$, (n is a positive integer)
$e^{at}\ \sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
$e^{at}\ \cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
$e^{at}\, f(t)$	$F(s-a)$
$\mathscr{U}(t-a)$	$\dfrac{e^{-as}}{s}, \quad a \geq 0$
$f(t-a)\mathscr{U}(t-a)$	$e^{-as}\, F(s)\ , \quad a \geq 0$
$g(t)\mathscr{U}(t-a)$	$e^{-as}\, \mathscr{L}\{g(t+a)\}\ , \quad a \geq 0$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - f^{(n-1)}(0)$
$\delta(t)$	1
$\delta(t-a)$	$e^{-as}\ , \quad a \geq 0$
$f(t)$ periodic with period T	$\dfrac{1}{1-e^{-sT}} \displaystyle\int_0^T e^{-st}\, f(t)\, dt$